Seat No. _______________

HAL-003-2015003

B. Sc. (Sem. V) (CBCS) (W.E.F.-2019) Examination

June – 2023

Mathematics : Paper-7(A)

(Boolean Algebra & Complex Analysis-I)

Faculty Code : 003
Subject Code : 2015003

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions :

(1) All questions are compulsory.
(2) Figure to the right indicate full marks of the questions.

1 (A) Answer the following questions : **4**

(1) Define : Anti-symmetric relation.
(2) Define : Cover of an element. Find cover(s) of 5 in S_{30}.
(3) Every complete lattice is bounded. [True/False]
(4) State De Morgan's laws for a complemented distributive lattice.

(B) Attempt any **one** : **2**

(1) Let $(L,\le)$ be a lattice. Prove that for any $a,b \in L, a \le b$ if and only if $a \oplus b = b$.
(2) Define : Order preserving map. Prove that every lattice homomorphism is order preserving map.

(C) Attempt any **one** : **3**

(1) Let $(L,\le)$ be a lattice. Prove that for any $a,b,c \in L$, if $b \le c$ then $a * b \le a * c$ and $a \oplus b \le a \oplus c$.
(2) Define : Complete lattice. Prove that every finite lattice is complete.

(D) Attempt any **one** : **5**

(1) Let $(L,*,\oplus)$ be a lattice. Prove that for any $a,b,c \in L, a \oplus (b * c) \le (a \oplus b) * (a \oplus c)$.
(2) Prove that every chain is distributive lattice.

2 (A) Answer the following questions : **4**

(1) $(S_{15},*,\oplus,',1,15)$ is a sub-boolean algebra of $(S_{30},*,\oplus,',1,30)$. [True/False]

(2) Define : Atom of a Boolean algebra.

(3) Define : Boolean homomorphism.

(4) How many maxterms can be formed using n variables ?

(B) Attempt any **one** : **2**

(1) Let $(B,*,\oplus,',0,1)$ be a boolean algebra. Prove that a nonempty subset S of B is subboolean algebra of B if S is closed under $*$ and '.

(2) Express the boolean expression $f(x_1,x_2,x_3)=x_1x_3+x_2x_3'$ as a sum of product canonical form.

(C) Attempt any **one** : **3**

(1) Let $(B,*\oplus,',0,1)$ and $(P,\wedge,\vee,^{-},\alpha,\beta)$ be two boolean algebra. Prove that if a mapping $f:B\to P$ preserves $*$ and ', then f is boolean homomorphism.

(2) Reduce the booleam expression $f(x_1,x_2,x_3)=x_1x_2'x_3+x_1'x_2x_3+x_1x_2+x_1'x_2'x_3$ using K-map.

(D) Attempt any **one** : **5**

(1) Define : Direct product of two boolean algebra. Prove that direct product of two boolean algebra is a boolean algebra.

(2) State and prove Stone's representation theorem.

3 (A) Answer the following questions. **4**

(1) Determine the points at which the function $f(z)=\dfrac{1}{x^3+z}$ is discontinous.

(2) Define : Analytic function.

(3) Define Simply connected domain.

(4) Show that $f(z)=z^2+1$ is entire function.

(B) Attempt any **one** : **2**

(1) Prove that if limit of a function of complex variable at point z_0 exists, then it is always unique.

(2) Check whether the function $f(z)-i\bar{z}^2$ is analytic at $z_0=x_0+iy_0(z_0\neq 0)$ or not.

(C) Attempt any **one** : **3**

(1) Prove that composition of two continuous functions of complex variable is continuous.

(2) Determine the analytic function $f(z) = u + iv$, where $u(x, y) = y^3 - 3x^2 y$.

(D) Attempt any **one** : **5**

(1) Show that the function

$$f(z) = \begin{cases} |z|^2; z \neq 0; \\ 0; \quad z = 0 \end{cases}$$

is differentiable only at origin.

(2) State and prove sufficient condition for a function of complex variable to be analytic at a point.

4 (A) Answer the following questions : **4**

(1) State Cauchy-Riemann equations in polar form.

(2) State Cauchy-Goursat theorem.

(3) If C represents the unit circle then $\int_C \frac{1}{z-2} dz =$ ________.

(4) If $C : |z - z_0| = r (r > 0)$ then $\int_C \frac{1}{z - z_0} dz =$ ________.

(B) Attempt any **one** : **2**

(1) Using Cauchy-Riemann equations in polar form, prove that the function $f(z) = \frac{1}{z} (z \neq 0)$ is analytic in its domain.

(2) If C is the arc of the circle |z| = 2 lying in the first quadrant, then find the upper bound of $\int_C \frac{1}{z^2 - 1} dz$.

(C) Attempt any **one** : **3**

(1) Evaluate $\int_{3i}^{2+4i} ((2y + x^2) + i(3x - y)) \, dz$ along $x = 2t, y = t^2 + 3$.

(2) If $f(z) = u(x, y) + iv(x, y)$ is an analytic function then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2$.

(D) Attempt any **one** : **5**

(1) Prove that $u(r,\theta) = r^2 \cos 2\theta$ is harmonic function. Find its harmonic conjugate and hence determine the analytic function $f(z) = u + iv$.

(2) State and prove Cauchy's integral formula.

5 (A) Answer the following questions : **4**

(1) Write Cauchy's extended integral formula for derivatives of an analytic function.

(2) State Cauchy's inequality.

(3) sin z is bounded. [True/False]

(4) What is the number of roots in $\mathbb{C}$ for the polynomial $z^7 + 1 = 0$?

(B) Attempt any **one** : **2**

(1) If C is unit circle, then evaluate $\int_C \frac{e^{2z}}{z^4} dz$.

(2) Evaluate $\int_C \frac{\cos hz}{z^3} dz$, where $C : |z| = 1$.

(C) Attempt any **one** : **3**

(1) If C is the circle $|z - i| = 2$, then evaluate

$$\int_C \frac{1}{(z^2+4)^2} dz.$$

(2) State and prove Liouville's theorem.

(D) Attempt any **one** : **5**

(1) If $f(z)$ is analytic within and on simple closed contour C and z_0 is any point interior to C, then prove that

$$f'(z_0) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z - z_0)^2} dz.$$

(2) State and prove fundamental theorem of algebra.